

Realized and Mediated Response Across Dynamical Domains: Minimal Models and Context-Dependent Thresholds

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Abstract

This paper continues the philosophical-physical framework developed in *Preface to the Trilogy on the Principle of Dynamical Sensitivity*, *The Principle of Dynamical Sensitivity of Structures within a Shared Physical Environment* (Paper I), *Operationalizing the Principle of Dynamical Sensitivity: Response, Coupling, and Cross-Scale Interaction* (Paper II), and *Mediated Interaction in a Shared Dynamical Environment: A Mechanistic Extension of the Principle of Dynamical Sensitivity* (Paper III). Its focus is a consequence already implicit in the earlier response-based formulation: the threshold of realized response need not always be treated as a single fixed criterion. [1–4]

The paper argues that thresholding should be treated in graded and context-dependent form, so that the same realized response may be evaluated differently depending on whether the relevant analytical aim concerns detectability, effective interaction, persistence, or structural consequence. On that basis, the paper develops a common abstract response skeleton and illustrates it in three connected minimal settings: a two-oscillator Kuramoto-type model, a candidate dynamical-network formulation, and a minimal mediated three-node chain.

Across these cases, the paper distinguishes inherited sensitivity as a guiding horizon, realized response as an operational category, and threshold-dependent effective regimes as analytically separable levels. In the mediated case, it further distinguishes latent channels from dynamically effective pathways and shows how graded thresholds may be extended to mediated realized response.

The paper does not propose new synchronization equations, a new universal response theorem, or a new physical force. Its contribution is instead a clarified cross-domain synthesis: inherited sensitivity, realized response, differentiated thresholding, and

mediated response are brought into one common analytical language equipped with minimal formal illustrations.

1. Introduction

The present paper continues the philosophical-physical synthetic framework developed in *Preface to the Trilogy on the Principle of Dynamical Sensitivity*, *The Principle of Dynamical Sensitivity of Structures within a Shared Physical Environment* (Paper I), *Operationalizing the Principle of Dynamical Sensitivity: Response, Coupling, and Cross-Scale Interaction* (Paper II), and *Mediated Interaction in a Shared Dynamical Environment: A Mechanistic Extension of the Principle of Dynamical Sensitivity* (Paper III). As stated in the Preface, these texts are intended to be read together as a single progression rather than as isolated claims, and the broader framework is proposed as a conceptual research program rather than as a claim of new physical laws, a new force, or a complete predictive theory. [1–4]

Paper I introduced The Principle of Dynamical Sensitivity as a conceptual proposal according to which structures emerging within a shared dynamical physical environment inherit sensitivity to its dynamical properties. In this sense, inherited sensitivity functions as the conceptual basis for the later distinction between latent responsiveness and threshold-dependent effective interaction. Paper II operationalized this proposal through a response-based framework distinguishing environmental excitation, system sensitivity, realized response, and threshold-dependent effective interaction. Paper III extended the same framework by addressing the possibility that realized response may become mediated, chained, stabilized, weakened, or fail through emergent intermediate dynamical regimes. [2–4]

The aim of the present paper is to continue this line by addressing a further consequence already implied in Paper II and reinforced in Paper III: namely, that the threshold of realized response need not always be treated as a single fixed criterion. Since Paper II explicitly states that the relevant band Ω and the threshold Θ depend on the class of system and on the practical level at which interaction is regarded as dynamically relevant, the present study explores whether threshold language should be generalized into a graded and context-dependent framework. On that basis, the paper offers minimal model illustrations of this language, most explicitly in the two-oscillator case and more tentatively in a dynamical network formulation, and proposes a corresponding extension toward mediated realized response. [3]

In Paper III, mediated interaction and chained response were introduced as mechanistic extensions of the response-based framework. In the present paper, these possibilities are gathered under the broader term *mediated realized response*, intended to denote any case in which realized response becomes dynamically accessible through an intermediate

regime, a medium, or an additional structural element rather than only through direct local response. In this sense, chained response is treated as one specific form of mediated realized response rather than as an independent category. [4]

The originality claimed in the present paper does not lie in proposing new synchronization equations, a new linear-response formalism, or a new physical force. Its more limited and more specific contribution lies in the explicit cross-domain synthesis of three elements: the inherited-sensitivity horizon introduced in the earlier texts, the response-based distinction between latent responsiveness and threshold-dependent effective interaction, and the graded or context-dependent specification of thresholds according to the intended analytical aim.

In this sense, the present work should be read as a philosophical-physical synthesis equipped with minimal formal illustrations rather than as a completed predictive theory. Its aim is to clarify a common analytical language by which different dynamical domains may be compared without erasing their disciplinary differences.

The novelty claimed here lies not in the existence of thresholds as such, but in the explicit treatment of threshold selection as part of the logic of evaluation itself, including differentiated thresholds tied to the intended analytical aim.

The contribution claimed in the present paper is therefore threefold: first, the clarification of inherited sensitivity as a guiding horizon of interpretation; second, the explicit cross-domain articulation of realized response as a common analytical language; and third, the graded and context-dependent treatment of threshold selection as part of the logic of evaluation itself.

2. Position of Paper IV Within the Sequence

Paper IV should be read as the next step in the structured progression established by the Preface and Papers I–III. Paper I introduced The Principle of Dynamical Sensitivity as a conceptual framework for interpreting structures formed within a shared dynamical physical environment. Paper II operationalized this framework through a response-based language centered on environmental excitation, system sensitivity, realized response, and threshold-dependent effective interaction. Paper III then extended the same line toward mediated interaction, chained response, and emergent intermediate dynamical regimes, thereby opening the mechanistic question of how realized response may persist, weaken, stabilize, or fail beyond the directly local case. [1–4]

Within that progression, the present paper addresses a further consequence already implicit in Paper II and reinforced in Paper III: namely, that the threshold of realized response need not always be treated as a single fixed criterion. If realized response can remain weak, transient, latent, measurable, effective, or structurally consequential

depending on the system and on the purpose of evaluation, then threshold language itself requires further refinement. Paper IV therefore extends the earlier framework by introducing graded and context-dependent thresholds and by asking how this refined threshold language may be illustrated in minimal model form. [3,4]

In this sense, Paper IV occupies an intermediate position between the operational language of Paper II and the more explicit model work that may follow later. It does not aim to provide a complete predictive theory or a final mechanistic derivation. Its role is narrower and more disciplined: to show, first, that the language of realized response and mediated realized response can be carried into minimal model examples, and second, that threshold-dependent interaction may need to be described in a graded and context-dependent way once one moves across domains or from direct to mediated regimes.

3. Recapitulation of the Existing Framework

Before extending the threshold framework in a graded and context-dependent direction, it is useful to restate the conceptual and operational basis established in the preceding texts. The present paper does not begin from an independent model-theoretical construction. It continues a framework already articulated across the Preface and Papers I–III, and therefore briefly recalls the elements on which the present extension depends most directly: The Principle of Dynamical Sensitivity, the distinction between realized response and threshold-dependent effective interaction, and the mechanistic opening toward mediated response. [1–4]

3.1 The Principle of Dynamical Sensitivity

The conceptual starting point of the framework is The Principle of Dynamical Sensitivity introduced in Paper I and clarified in the Preface. Its central proposal is that structures emerging within a shared dynamical physical environment inherit sensitivity to its dynamical properties. This inherited sensitivity is not presented as a new force or as a completed mechanism of interaction, but as a working hypothesis concerning the common dynamical embedding of structures formed within the same broader background. [1,2]

Within this perspective, dynamical sensitivity is more general than observable interaction. A system may remain sensitive in principle to the dynamical structure of its environment even when no effective resonance, coupling, synchronization, or cross-scale interaction becomes observable. This distinction is essential for the trilogy as a whole, because it allows inherited sensitivity to remain conceptually present below the level at which realized response becomes dynamically effective.

3.2 Realized Response and Threshold-Dependent Effective Interaction

Paper II translated the conceptual language of sensitivity into an operational response-based framework. In this formulation, environmental excitation, intrinsic system sensitivity, and realized response are distinguished analytically rather than collapsed into a single notion of interaction. The realized response is written as the product of excitation and sensitivity, and dynamically significant interaction is expressed through an integrated influence exceeding a threshold. [3]

The importance of this formulation lies in the explicit statement that the threshold is not a boundary of sensitivity itself. A system may remain dynamically responsive in principle while the realized response stays below the level required for effective interaction, observable resonance, synchronization, or other higher-order regimes. The threshold therefore separates latent responsiveness from dynamically effective interaction, not sensitivity from non-sensitivity. This distinction forms one of the architectural cores of the trilogy and is the immediate conceptual basis of the present paper.

3.3 Mediated and Chained Response

Paper III extended the earlier framework by asking how realized response may persist, weaken, stabilize, or fail once one moves beyond the directly local case. Its main contribution was to open the possibility that realized response need not remain only a transient local event, but may under suitable conditions become mediated through intermediate regimes embedded in the same shared dynamical environment. In this way, the framework moved from direct response alone toward the possibility of mediated organization. [4]

At the same time, Paper III made clear that mediation is not automatically effective. Intermediate pathways may remain weak, transient, or sub-threshold; not every chain becomes dynamically effective, and not every compatible relation persists. This clarification is decisive for the present paper, because it shows that threshold language must also be capable of describing mediated regimes at different levels of weakness, persistence, and organization. The present study therefore treats mediated realized response as a natural extension of the earlier response-based framework rather than as a separate category outside it.

4. From a Single Threshold to Graded and Context-Dependent Thresholds

In Paper II, the threshold of realized response was introduced in the form $I_{\Omega} > \Theta$, with the explicit clarification that neither the relevant band Ω nor the threshold Θ is universal. Both depend on the class of system under consideration and on the practical level at

which interaction is regarded as dynamically relevant. The present paper takes this implication as its starting point and argues that, once realized response is considered across different domains and across both direct and mediated regimes, threshold language should no longer be treated as necessarily single-level. [3]

This shift does not alter the basic architecture of the earlier framework. The distinction between intrinsic sensitivity, realized response, and threshold-dependent effective interaction remains unchanged. What changes is the way in which threshold significance is specified. Instead of assuming a single fixed boundary between “non-effective” and “effective” response, the present paper proposes that different analytical aims may legitimately require different threshold criteria, provided that these criteria are stated explicitly before the calculation and remain tied to a clearly defined scientific purpose.

4.1 Functional and Empirical Thresholds

The threshold of realized response should therefore be understood as functional or empirical rather than absolute. A threshold may be introduced in order to mark detectability, effective interaction, persistence, or structural relevance, depending on what the given calculation is intended to show. In this sense, the threshold does not define whether sensitivity exists in principle; it defines when a realized response becomes relevant for the particular analytical purpose under consideration.

Such an interpretation is especially important in systems that do not naturally separate into only two observable regimes. In biology, a weak response may already be measurable without yet becoming functionally decisive. In a dynamical network, one threshold may be chosen for a detectable output shift, another for a meaningful change of weights, and another for a more structural reorganization of the network. In chemistry, similarly, one may distinguish between a measurable dynamical reaction, an effectively sustained regime, and a more consequential pattern-forming transition. The point is not to multiply thresholds arbitrarily, but to preserve one coherent response-language while allowing its criterion of relevance to follow the actual purpose of the analysis.

4.2 Graded Thresholds

Once threshold choice is understood in this way, the single-threshold criterion may be generalized into a graded structure. A realized response may be sub-threshold with respect to one level of evaluation and yet above threshold with respect to another. This allows one to distinguish, for example, between measurable response, dynamically effective response, persistent response, and structurally consequential response without abandoning the same underlying analytical framework.

This graded approach is particularly useful when moving from direct to mediated response. A mediated pathway may be measurable without yet being dynamically effective; it may be dynamically effective without yet being persistent; and it may remain persistent without necessarily producing broader structural reorganization. In this way, graded thresholds allow the same language to describe not only whether response occurs, but also at what level it becomes scientifically meaningful.

4.3 Context-Dependent Thresholds

The most compact way to express this extension is to let the threshold depend explicitly on the context of evaluation. The relevant criterion is then not simply whether I exceeds some universal threshold, but whether it exceeds the threshold appropriate to the intended analytical context. Such a context may refer to measurement, function, persistence, reorganization, or any other explicitly defined level of scientific relevance.

Under this formulation, the originality does not lie in the existence of thresholds as such, since threshold criteria are already well known in many fields. The contribution lies rather in making explicit that the threshold of realized response belongs to the logic of the analysis itself and should therefore be specified in relation to what the calculation is intended to establish. This makes it possible to preserve one response-based language across domains while still respecting the fact that different domains and different scientific questions require different criteria of relevance.

4.4 A Common Abstract Skeleton and Its Domain-Specific Instantiations

Before turning to the minimal model examples themselves, it is useful to state explicitly the common abstract skeleton that underlies the present comparison. The aim of this skeleton is not to erase disciplinary differences, but to identify the minimal analytical structure that remains comparable across them. In this sense, the framework begins not from a claim that Kuramoto oscillators and dynamical networks are identical systems, but from the more modest claim that both may be read through a common response-based architecture.

In its most abstract form, this skeleton contains five elements: a system state, an excitation or dynamical influence, a structured sensitivity or transfer profile, a realized response, and a threshold of relevance or effectiveness. The specific physical meaning of these elements may vary from one domain to another, but the analytical relations among them remain comparable.

$$R = \chi e$$

$$R(t) = \int G(t, s) e(s) ds$$

$$R(\omega) = \chi(\omega)e(\omega)$$

These expressions should not be read as a demand that every model in the present paper take exactly the same formal shape. Their role is more limited: they define the abstract architecture within which realized response may be understood as the effect generated when an excitation is filtered by a structured susceptibility or transfer relation. This is the point at which the operational language of Paper II meets the broader comparative aim of the present paper.

In order to determine whether response remains merely latent or becomes dynamically relevant, the framework then introduces a measure of realized response and a threshold condition. Depending on the domain, this measure may be expressed in frequency space, in continuous time, or in discrete time, but its role remains the same: to distinguish mere possibility or weak responsiveness from dynamically effective realization.

$$I_{\Omega} = \int_{\Omega} |R(\omega)|^2 d\omega$$

$$I_T = \frac{1}{T} \int_0^T \|R(t)\|^2 dt$$

$$I > \Theta$$

Within this common skeleton, the two-oscillator model of Section 5 and the dynamical network model of Section 6 can be understood as domain-specific instantiations rather than as formally identical realizations. In the two-oscillator case, the state is represented by the phase difference, the internal separating tendency by the detuning, and the realized response by the coupling-generated phase correction. In the dynamical network case, the state is represented by the evolving internal configuration, the excitation by input perturbation, and the realized response by induced changes of state, output, or adaptive parameters.

What makes the comparison legitimate is therefore not identity of mechanism, but comparability of structure. Both models permit a distinction between underlying inherited sensitivity or predisposition, an actually realized response under given conditions, and a further threshold-dependent criterion separating weak or latent responsiveness from effective realization. This is the precise sense in which the present paper speaks of minimal models across dynamical domains.

This common skeleton also permits a differentiated threshold structure. The same realized response measure need not be evaluated against only one fixed criterion of relevance. Instead, a family of thresholds may be introduced whenever the same response is to be read at different analytical levels, such as detectability, effective interaction, persistence, or structural reorganization.

In this case, the role of the threshold is not to redefine the response itself, but to specify the level at which that response is to count as scientifically relevant for the intended calculation. The differentiated threshold is therefore not an arbitrary addition to the model, but a formal expression of the fact that one and the same realized response may be interpreted under different, explicitly stated evaluation contexts.

$$\Theta^{(1)} < \Theta^{(2)} < \Theta^{(3)} < \dots$$

$$I > \Theta(\mathcal{C})$$

This formulation makes explicit why the same response-based skeleton can be carried across domains without imposing a single universal cutoff. What varies is not the existence of realized response, but the criterion by which that response is judged to be measurable, effective, persistent, or reorganizationally significant.

The response expressions introduced in the abstract skeleton should therefore be read as comparative response templates rather than as a claim that all systems considered here obey one and the same derived response theory in a strict sense. Their role is to provide a disciplined analytical language for relating excitation, structured sensitivity, realized response, and thresholded relevance across domains.

In particular, the use of Kubo-like or susceptibility-based notation in the present paper should be understood as a local, comparative, and structurally illuminating formal analogy. It is not intended to imply that every nonlinear, adaptive, or mediated system considered here is already covered by a single universal response theorem. [5]

5. Minimal Explicit Instantiation: Two Coupled Oscillators

The first minimal model considered in the present paper is the case of two coupled oscillators. This choice is motivated by analytical clarity: the two-oscillator case is simple enough to remain transparent, yet already rich enough to display the central distinction between latent responsiveness and dynamically effective coordination.

The role of this section is not to prove the broader framework in full, but to provide a first explicit model illustration of thresholded realized response. In this sense, the two-oscillator case is introduced as a disciplined pilot instantiation of the response-based language rather than as an exhaustive realization of the philosophical-physical framework as a whole.

5.1 Kuramoto-Type Formulation

As a minimal nonlinear model, consider two phase oscillators with intrinsic frequencies ω_1 and ω_2 , coupled symmetrically with strength K . Their dynamics may be written in the standard Kuramoto-type form [6,7]

$$\dot{\theta}_1 = \omega_1 + K \sin(\theta_2 - \theta_1)$$

$$\dot{\theta}_2 = \omega_2 + K \sin(\theta_1 - \theta_2)$$

where θ_1 and θ_2 denote the phases of the two oscillators. In this formulation, the intrinsic frequencies express the internal dynamical tendencies of the two units, while the coupling term expresses the structured dynamical influence that each oscillator exerts on the other within the shared local environment of the model.

To obtain a more transparent representation of the dynamics, introduce the phase difference

$$\phi = \theta_2 - \theta_1$$

and the frequency mismatch

$$\Delta\omega = \omega_2 - \omega_1$$

so that the system reduces to a single equation for the phase difference:

$$\dot{\phi} = \Delta\omega - 2K \sin \phi$$

This reduced form is especially useful because it makes the competition between detuning and coupling explicit. The difference between the intrinsic frequencies tends to separate the oscillators, while the coupling tends to organize their phases into a more coordinated regime.

At this stage, the model is introduced only as a minimal dynamical setting. The explicit interpretation of realized response and the corresponding threshold condition will be developed in the following subsection.

5.2 Realized Response and Threshold Condition

In order to connect the two-oscillator model with the response-based language of Paper II, it is necessary to distinguish between the intrinsic dynamical tendency of each oscillator and the dynamical modification imposed by coupling. In the absence of coupling, each oscillator evolves according to its own intrinsic frequency. Once coupling is introduced, the interaction term represents the realized dynamical response produced under the actual relative phase configuration of the pair.

On this basis, the local realized responses of the two oscillators may be written as the coupling contributions appearing in their equations of motion. These terms do not express sensitivity in itself. Rather, they express the actually realized effect of the shared dynamical configuration on the evolution of each oscillator at a given moment.

$$R_1(t) := K \sin(\theta_2 - \theta_1)$$

$$R_2(t) := K \sin(\theta_1 - \theta_2)$$

In the reduced phase-difference formulation, the same idea may be expressed more compactly by identifying the coupling-driven part of the phase-difference dynamics as the effective realized response acting against detuning. In this minimal case, the

competition is therefore not between sensitivity and non-sensitivity, but between detuning and the realized response generated by coupling.

$$R_\phi(t) := 2K \sin \phi$$

The threshold condition appears when one asks under what circumstances the realized response is strong enough to balance the intrinsic tendency toward phase separation. A phase-locked state requires a stationary phase difference, that is, $\dot{\phi} = 0$. Under this condition, the reduced equation yields a balance relation between detuning and coupling-driven response.

$$\Delta\omega = 2K \sin \phi$$

Since $\sin \phi$ is bounded in magnitude by 1, a phase-locked solution exists only if the realized response can in principle compensate the frequency mismatch. This yields the explicit threshold condition

$$|\Delta\omega| \leq 2K$$

This condition provides the first minimal explicit threshold of realized response within the present paper. If $K > 0$ but $|\Delta\omega| > 2K$, then coupling is present and the oscillators are not dynamically isolated, yet the realized response is not sufficient to stabilize effective coordination. By contrast, when $|\Delta\omega| \leq 2K$, the coupling-generated response is strong enough for the pair to enter a phase-locked regime.

In this way, the standard two-oscillator model becomes readable in the language of the trilogy: non-zero coupling indicates a dynamical predisposition to interaction and may be read, in minimal model form, as the analogue of inherited sensitivity; realized response expresses the actual dynamical effect of coupling; and the threshold condition separates latent responsiveness from dynamically effective coordination. The more general integrated threshold language of Paper II is therefore not abandoned here; rather, in this minimal case it takes the analytically explicit form of a locking condition.

5.3 Latent Responsiveness and Effective Coordination

The two-oscillator case now makes it possible to state the distinction between latent responsiveness and effective coordination in explicitly dynamical terms. The crucial point is that non-zero coupling does not by itself imply the emergence of a coordinated regime. It indicates only that the oscillators are no longer dynamically isolated and that a realized response may occur under the given phase relation.

In the language of the trilogy, this corresponds to a minimal model situation in which dynamical predisposition and actual response are already present, while dynamically effective coordination has not yet been achieved. The model therefore preserves the distinction between the possibility of interaction and the emergence of an effective regime.

$$K > 0$$

$$|\Delta\omega| > 2K$$

When the coupling is non-zero but the detuning remains too large, the oscillators influence one another without stabilizing into a phase-locked state. In this regime, realized response is present, but it remains sub-threshold with respect to effective coordination. This is the minimal explicit form of latent responsiveness within the present model.

Such a regime is important because it shows that the absence of locking does not imply the absence of sensitivity or even the absence of response. What is absent is only the higher-order stabilization required for dynamically effective coordination. In this respect, the model directly supports the distinction already emphasized in Paper II between latent responsiveness and threshold-dependent effective interaction.

$$|\Delta\omega| \leq 2K$$

By contrast, when the threshold condition is satisfied, the realized response becomes strong enough to compensate the detuning and to stabilize a coordinated phase relation. At that point, the same system moves from mere dynamical predisposition and realized response into a dynamically effective regime. Effective coordination is therefore not the

presence of coupling itself, but the threshold-dependent stabilization of the response generated by coupling.

The value of this minimal model lies precisely here. It shows, in analytically transparent form, that one may distinguish between sensitivity, realized response, and effective coordination without introducing different ontological categories for each regime. Instead, these regimes can be read as different dynamical levels of one and the same response structure. This is exactly the kind of distinction that the present paper will later extend toward context-dependent thresholds and mediated realized response.

6. Candidate Extension by Analogy: Dynamical Network Formulation

The two-oscillator case provides a minimal explicit instantiation of thresholded realized response in a familiar nonlinear dynamical setting. The next step is not to replace that model, but to ask whether the same analytical language may be carried, in a more tentative way, into a dynamical network formulation.

This section is therefore introduced explicitly as a candidate extension by analogy rather than as an already established result of the framework. Its aim is to show that the distinction between sensitivity, realized response, threshold-dependent relevance, and later mediated response may also be formulated in a system whose dynamics is not described primarily in terms of phase locking, but in terms of state evolution and possible internal reconfiguration.

6.1 Minimal State-Space Description

As a minimal candidate model, consider a dynamical system whose internal state evolves in discrete time under the influence of its previous state and an external or environmental input. Let h_t denote the internal state at time t , u_t the input, and y_t the observable output. In its most general form, such a system may be written as

$$h_{t+1} = F(h_t, u_t)$$

$$y_t = G(h_t)$$

where F governs state evolution and G maps the internal state to an observable output. This formulation is intentionally broad. It does not yet assume a particular learning rule, architecture, or application domain. Its only purpose at this stage is to provide a minimal dynamical setting in which sensitivity, realized response, and threshold-dependent relevance may be distinguished in a way analogous to the two-oscillator case.

In order to make this analogy more explicit, one may consider a local linearization around a reference trajectory or operating regime. In that case, small perturbations of the state and input may be written in the form

$$\begin{aligned}\delta h_{t+1} &= A_t \delta h_t + B_t \delta u_t \\ \delta y_t &= C_t \delta h_t\end{aligned}$$

In this linearized form, the analogy with the response-based language of Paper II becomes clearer. The perturbation δu_t plays the role of an excitation, the matrices A_t , B_t , and C_t express the structured dynamical sensitivity of the system in the given regime, and the induced changes δh_t and δy_t provide natural candidates for realized response.

In this sense, the linearized network description is introduced as a response-compatible local approximation, not as a claim that the full nonlinear network dynamics is exhausted by this perturbative form. The analogy is strongest at the level of analytical structure, and more limited at the level of exact mechanism.

At this stage, no single choice is imposed as universally correct. Depending on the analytical purpose, realized response may later be read at the level of state change, output change, weight change, or more structural network reorganization. For the moment, however, the essential point is simpler: even in this broader dynamical setting, one may preserve the same basic distinction between intrinsic sensitivity, actual realized response, and a further threshold-dependent criterion of scientific relevance.

6.2 Graded Thresholds in a Dynamical Network

Once the state-space formulation is introduced, the threshold framework may be extended in a way that makes the context of evaluation explicit. In a dynamical network, realized response need not be read at only one level. Depending on the purpose of the analysis, the relevant response may be sought in the internal state, in the observable output, in adaptive parameters, or in more structural forms of reorganization.

For this reason, a single threshold is often insufficient. A weak perturbation may be detectable in the state trajectory without yet producing a meaningful output change. An output shift may be measurable without yet implying adaptation of weights or couplings. And even adaptive change may remain local or transient without amounting to broader network reorganization. In such cases, the distinction introduced in Section 4 becomes operationally useful: the same response-based language can be preserved while the threshold is specified according to the intended analytical level.

A first natural candidate is the state-response level. If the relevant question concerns the internal reaction of the network to perturbation, one may define an integrated state-response measure over a finite time horizon T .

$$I_h(T) := \frac{1}{T} \sum_{t=0}^{T-1} \|\delta h_t\|^2$$

The corresponding threshold criterion then takes the form

$$I_h(T) > \Theta_h$$

In this case, the threshold Θ_h expresses the level above which internal state variation is regarded as analytically relevant. Such a threshold does not yet imply that the network has changed its observable behavior; it identifies only that the perturbation has produced a sufficiently strong realized response at the level of internal dynamics.

A second natural level is the output-response level. If the relevant question concerns what the network actually expresses outwardly, the same logic may be applied to the output variable.

$$I_y(T) := \frac{1}{T} \sum_{t=0}^{T-1} \|\delta y_t\|^2$$

$$I_y(T) > \Theta_y$$

Here, the threshold Θ_y expresses the level above which the realized response becomes relevant at the level of observable output. The same perturbation may therefore remain below threshold with respect to output while already lying above threshold with respect to the internal state. This illustrates directly why graded thresholding is not merely optional but often necessary in network-like systems.

In adaptive network settings, one may also consider thresholds defined with respect to changes in parameters such as weights or couplings. If the network includes adaptive variables W_t , then realized response may be examined not only through state and output, but also through the magnitude of parameter update.

$$I_W(T) := \frac{1}{T} \sum_{t=0}^{T-1} \|\delta W_t\|^2$$

$$I_W(T) > \Theta_W$$

In this form, the threshold Θ_W marks the point above which parameter adaptation itself becomes analytically relevant. This is especially important for systems in which the response is not exhausted by momentary state fluctuation, but leaves a trace in the evolving internal architecture of the network.

Finally, one may distinguish a still stronger level corresponding to structural or organizational reconfiguration. This level is deliberately left more schematic here, because a general metric of reorganization depends strongly on the class of network under consideration. Nevertheless, the principle remains the same: the threshold is not imposed as a universal constant, but chosen according to the scientific question being asked.

In this way, the dynamical network formulation makes explicit what was only implicit in the earlier threshold language: realized response may be measurable at one level, effective at another, and structurally consequential at a third. The originality does not lie in inventing thresholds as such, but in making clear that the threshold of realized response belongs to the logic of the calculation itself. Different thresholds are therefore legitimate when they correspond to different, explicitly stated analytical aims within one coherent response-based framework.

6.3 Limits of the Analogy

The analogy developed in the present section is intended to be structurally illuminating rather than mechanically exhaustive. Its purpose is to show that the response-based language introduced in Paper II and extended in Paper III may be carried into a dynamical network setting in a disciplined way. It is not intended to claim that a dynamical network and a pair of coupled oscillators are governed by the same mechanism in any strong or literal sense.

The two-oscillator model offers an explicit threshold relation in analytically closed form. In the network case, by contrast, thresholded realized response is described through candidate measures of state variation, output variation, and adaptive change. These measures are formally comparable to the earlier threshold language, but they do not yet yield a unique universal criterion of effective coordination analogous to phase locking.

For this reason, the analogy should be read as one of structural correspondence rather than one of complete mathematical identity. What is shared is the distinction between underlying sensitivity, realized response under concrete conditions, and a threshold-dependent transition to a more effective regime. What is not shared in identical form is the detailed mechanism through which such a transition is produced, stabilized, or measured.

A further limit concerns the choice of threshold itself. In the oscillator case, the threshold condition emerges directly from the dynamics in closed form. In the dynamical network case, the threshold is intentionally tied to the analytical purpose of the calculation. This is not a weakness of the framework, but it does mean that network thresholds remain context-dependent and partly model-class dependent. Their legitimacy therefore depends on whether the chosen criterion is stated explicitly and justified by the scientific question being asked.

Finally, the present section does not yet provide a full theory of learning, adaptation, or structural reorganization in network systems. It introduces only a minimal analytical bridge showing how such phenomena may be read within the language of realized response. Stronger claims would require a later stage of work involving more specific architectures, explicit adaptation rules, and possibly numerical or empirical validation.

These limits are important because they preserve the intended level of the paper. The present work does not claim a universal network formalism derived from first principles, but a minimal transdomain extension of the response-based framework. In this sense, the analogy remains strong enough to be useful, yet sufficiently bounded to remain methodologically honest.

7. Extension to Mediated and Chained Response

The preceding sections established the response-based language first in a direct minimal model and then in a broader dynamical-network analogy. The next step is to ask how the same framework may be extended once realized response is no longer treated only as a directly local event, but as something that may become dynamically accessible through an intermediate regime, a medium, or an additional structural element.

In this sense, the present section does not introduce a new category outside the earlier framework. It develops what Paper III had already opened: the possibility that realized response may become mediated, chained, stabilized, weakened, or fail depending on the organization of intermediate relations. The purpose here is therefore not to claim a complete mechanistic theory of mediation, but to provide a first minimal formal sketch of how mediated realized response may be described within the same analytical language.

Within this perspective, chained response is treated as one specific configuration of mediated realized response. What matters is not merely that several local interactions coexist, but that they form a pathway along which realized response may be carried beyond the immediately local relation. The problem is therefore no longer only whether response occurs, but whether it can be mediated across an ordered intermediate structure.

The mediated configurations discussed in the present section should not be understood as replacing or renaming the already established literature on relay, remote, or other forms of mediated synchronization. Rather, the present framework seeks to provide a broader response-based language within which such configurations may be interpreted as particular organized cases of mediated realized response. [8–10]

In this sense, the aim of the present section is not to claim priority over the known phenomena themselves, but to clarify how they may be read within a graded framework distinguishing latent channels, measurable mediated pathways, dynamically effective pathways, and more persistent organized regimes. [8–10]

7.1 Minimal Three-Node Chain

As a first minimal formal sketch, consider three units or nodes arranged in an ordered chain $1 \rightarrow 2 \rightarrow 3$. The first unit acts on the second, the second on the third, while the direct influence of the first on the third is assumed to be absent, negligible, or below the relevant direct threshold. In this way, any effective relation between the first and the third unit must be understood, if present, as mediated rather than direct.

$$R_{2\leftarrow 1}(t)$$

$$R_{3\leftarrow 2}(t)$$

$$R_{3\leftarrow 1}^{\text{dir}}(t)$$

Let $R_{2\leftarrow 1}(t)$ denote the local realized response of unit 2 to unit 1, and $R_{3\leftarrow 2}(t)$ the local realized response of unit 3 to unit 2. If needed for comparison, let $R_{3\leftarrow 1}^{\text{dir}}(t)$ denote the direct response of unit 3 to unit 1. The chain structure becomes analytically meaningful only when the direct pathway remains sufficiently weak relative to the mediated one.

$$I_{3\leftarrow 1}^{\text{dir}}(T) := \frac{1}{T} \int_{t_0}^{t_0+T} |R_{3\leftarrow 1}^{\text{dir}}(t)| dt$$

$$I_{3\leftarrow 1}^{\text{dir}}(T) \leq \Theta_{\text{dir}}$$

A minimal way to characterize the mediated pathway is then to define a chain indicator that depends on the co-presence of the two local realized responses. If either local response vanishes, the mediated chain cannot be dynamically sustained. If both are present over a finite interval, the possibility of mediated realized response becomes analytically meaningful.

$$I_{\text{chain}}(T) := \frac{1}{T} \int_{t_0}^{t_0+T} |R_{2\leftarrow 1}(t) R_{3\leftarrow 2}(t)| dt$$

This quantity is intentionally minimal and should not yet be read as a complete theory of mediation. It should therefore be understood as one possible minimal composite indicator of mediated realized response rather than as a unique canonical measure of mediation. Its role is narrower: it provides a compact measure of whether the chain possesses nontrivial mediated response over the interval under consideration. It is therefore a first analytical candidate for mediated realized response in a configuration where the final unit does not respond primarily by direct exposure to the first, but through an ordered intermediate relation.

In this form, chained response appears not as a new ontological category, but as a specific organized regime within the broader class of mediated realized response. The conceptual gain is the same as in the earlier sections: one may distinguish between the mere possibility of such a pathway, its measurable presence, and its later transition into a more effective or persistent mediated regime. The graded thresholds appropriate to that distinction will be introduced in the following subsection.

7.2 Graded Thresholds for Mediated Response

Once a minimal chain indicator has been introduced, the next step is to formulate the threshold structure appropriate to mediated response. The central point is the same as in the earlier sections: mediated realized response should not be read through only one fixed binary boundary. A mediated pathway may be merely measurable, dynamically effective, or sufficiently stable to count as a persistent organized regime.

This extension follows directly from the logic already established in Paper II and Paper III. Paper II treats the threshold as functional or empirical rather than universal, while Paper III makes clear that mediated pathways may differ in weakness, persistence, and organization. The present subsection therefore introduces not a new logic, but a more explicit threshold structure for a case in which mediation itself may exist in several analytically distinct regimes.

$$\Theta_{\text{chain}}^{\text{meas}} < \Theta_{\text{chain}}^{\text{eff}} < \Theta_{\text{chain}}^{\text{persist}}$$

Under this graded formulation, the same chain indicator may be interpreted at several levels. A first threshold may mark the level at which mediated response is measurably present. A second threshold may mark the level at which the mediated chain becomes dynamically effective. A third, stronger threshold may provisionally mark the level at which the mediated chain is sufficiently stable and organized to count as persistent.

$$I_{\text{chain}}(T) > \Theta_{\text{chain}}^{\text{meas}}$$

$$I_{\text{chain}}(T) > \Theta_{\text{chain}}^{\text{eff}}$$

$$I_{\text{chain}}(T) > \Theta_{\text{chain}}^{\text{persist}}$$

These inequalities should be read under the assumption introduced in the previous subsection, namely that the direct pathway from unit 1 to unit 3 remains absent, negligible, or below the relevant direct threshold. Under that condition, the graded chain thresholds do not merely quantify the strength of a local effect, but characterize the mediated pathway as such.

$$I_{3 \leftarrow 1}^{\text{dir}}(T) \leq \Theta_{\text{dir}}$$

The first threshold, $\Theta_{\text{chain}}^{\text{meas}}$, indicates that the mediated pathway is present at a measurable level. The second, $\Theta_{\text{chain}}^{\text{eff}}$, indicates that the mediated pathway is no longer merely detectable but strong enough to count as dynamically effective within the intended analysis. The third, $\Theta_{\text{chain}}^{\text{persist}}$, is introduced more cautiously: in the present paper it should be read as a first operational proxy for persistence, not yet as a complete temporal stability criterion.

This restriction is mathematically important. A large integrated chain indicator may, in principle, result either from sustained mediated response or from a shorter but stronger burst of mediated activity. For this reason, the persistence threshold introduced here should be understood as a minimal analytical placeholder whose fuller justification would require additional temporal or structural criteria in a later stage of the framework.

A compact way to express the same idea is to introduce a context-indexed chain threshold. In that case, the threshold is tied not to a fictional universal cutoff, but to the analytical purpose under which the mediated pathway is being evaluated.

$$I_{\text{chain}}(T) > \Theta_{\text{chain}}(\mathcal{C})$$

Here, $\mathcal{C}$ may denote detectability, effectiveness, persistence, or some domain-specific criterion of mediated relevance. In this form, the mediated-response framework becomes fully consistent with the graded threshold logic already introduced for direct response and for dynamical networks. The same analytical language is preserved, while the criterion of relevance is allowed to vary with the intended level of interpretation.

7.3 Latent Channels and Dynamically Effective Pathways

The distinction introduced in the preceding subsections makes it possible to separate the existence of a mediated channel from the emergence of a dynamically effective mediated pathway. This distinction is essential for the present framework, because Paper III already implies that mediation may remain weak, transient, or sub-threshold without thereby disappearing as a structured possibility.

In this sense, a latent channel should not be understood as the complete absence of mediation, but as a regime in which the intermediate relations are present and analytically meaningful, while the end-to-end mediated effect has not yet become sufficiently strong to count as dynamically effective. The chain therefore exists as a structured potential pathway before it necessarily appears as an effective one.

A minimal way to formalize this distinction is to introduce local link measures for the two elementary mediated relations.

$$I_{2 \leftarrow 1}(T) := \frac{1}{T} \int_{t_0}^{t_0+T} |R_{2 \leftarrow 1}(t)| \, dt$$

$$I_{3 \leftarrow 2}(T) := \frac{1}{T} \int_{t_0}^{t_0+T} |R_{3 \leftarrow 2}(t)| \, dt$$

If both local link measures exceed their respective local detectability thresholds, and if the direct pathway from 1 to 3 remains below the relevant direct threshold, then the system contains a latent mediated channel in the minimal sense intended here.

$$I_{2 \leftarrow 1}(T) > \Theta_{2 \leftarrow 1}^{\text{meas}}$$

$$I_{3 \leftarrow 2}(T) > \Theta_{3 \leftarrow 2}^{\text{meas}}$$

$$I_{3 \leftarrow 1}^{\text{dir}}(T) \leq \Theta_{\text{dir}}$$

These conditions do not yet imply that the mediated pathway is dynamically effective from the first unit to the third. They establish only that the chain possesses a structured

intermediate route along which mediated realized response may, in principle, be carried. The end-to-end mediated effect still requires a further criterion.

That further criterion is given by the graded chain thresholds introduced in the previous subsection. A latent channel becomes a measurably present mediated pathway when the chain indicator exceeds the measurable chain threshold, and it becomes a dynamically effective pathway when it exceeds the effective chain threshold.

$$I_{\text{chain}}(T) > \Theta_{\text{chain}}^{\text{meas}}$$

$$I_{\text{chain}}(T) > \Theta_{\text{chain}}^{\text{eff}}$$

The conceptual importance of this distinction is substantial. It shows that the absence of an effective mediated pathway does not imply the absence of mediating structure. A system may already contain organized local relations sufficient to define a latent channel, while the integrated mediated response remains below the threshold required for effective end-to-end propagation.

This is the mediated analogue of the distinction already established earlier between latent responsiveness and threshold-dependent effective interaction. In the direct case, a system may remain dynamically responsive without entering a coordinated regime. In the mediated case, a chain may remain structurally and dynamically prepared without yet becoming an effective transmitted pathway.

If a stronger criterion is later imposed, for example one tied to persistence or organized stability, the same logic remains applicable. The key point is unchanged: mediated response may exist at several analytically distinct levels, and the framework gains precision by separating latent channels from effective pathways rather than collapsing them into a single binary condition.

In this way, the mediated extension preserves the central discipline of the trilogy. What varies from regime to regime is not the existence of one completely different kind of phenomenon, but the degree to which one and the same response structure becomes measurable, effective, persistent, or organizationally significant.

8. Cross-Domain Illustrations

The purpose of the present section is not to introduce a further model class, but to show more explicitly how the response-based language developed in the preceding sections may be read across different dynamical domains. What matters here is not identity of mechanism, but comparability of analytical structure: a system may possess underlying inherited sensitivity or predisposition, may exhibit realized response under concrete conditions, and may cross different thresholds of relevance depending on the level at which the response is being evaluated.

In this sense, the following illustrations should be read as comparative uses of one framework rather than as exhaustive domain theories. Their purpose is to make visible how the same distinction between latent responsiveness, realized response, and threshold-dependent effective regime may remain meaningful when one moves from direct oscillatory interaction to network dynamics and then further to mediated pathway organization.

$$I > \Theta(\mathcal{C})$$

The compact criterion above expresses the common evaluative logic of the paper. The same realized response need not be judged by one universal boundary, but by the threshold appropriate to the interpretive context $\mathcal{C}$. In this way, the cross-domain comparison remains disciplined: what changes from case to case is not the existence of response itself, but the criterion by which response is treated as measurable, effective, persistent, or structurally consequential.

In the two-oscillator case, the distinction is especially transparent. Non-zero coupling indicates that the oscillators are not dynamically isolated, yet this alone does not suffice for effective coordination. The realized response generated by coupling becomes dynamically effective only when it is strong enough to compensate detuning. The threshold therefore separates latent responsiveness from phase-locked coordination rather than sensitivity from non-sensitivity.

In the dynamical-network case, the same logic becomes multi-level. A perturbation may produce a detectable state change without yet yielding a relevant output change, and an output change may in turn occur without implying significant adaptation or reorganization. Here the differentiated threshold structure becomes especially useful, because it allows the same realized response to be interpreted at the level of state, output, adaptive parameter change, or broader structural consequence without forcing these levels into one binary scheme.

In the mediated case, the framework gains a further degree of articulation. A chain may contain local intermediate relations sufficient to define a latent channel while still failing to produce an effective end-to-end pathway. This makes it possible to distinguish not only between direct and mediated response, but also between latent mediated structure, measurable mediated presence, dynamically effective mediated propagation, and more persistent organized mediated regimes.

These examples also indicate why the present framework may be of interest beyond the specific models explicitly developed here. In biological systems, chemical regimes, and other complex dynamical settings, it is often insufficient to ask only whether interaction is present or absent. What matters instead is whether the same response remains weak, measurable, functionally effective, persistent, or reorganizationally significant under the intended analytical aim.

For this reason, the cross-domain value of the present approach lies less in replacing established domain theories than in clarifying a common evaluative and response-based language. The same response structure may then be compared across systems without pretending that all such systems are mathematically identical or governed by one and the same mechanism.

9. Discussion and Limitations

The present paper has argued that the response-based framework introduced in Paper II and extended in Paper III becomes more precise when the threshold of realized response is no longer treated as necessarily single-level. Across the minimal examples developed here, the same analytical distinction has remained stable: a system may bear an underlying, inherited sensitivity or predisposition, may exhibit realized response under concrete conditions, and may cross different thresholds depending on the level at which the response is being evaluated.

On that basis, the paper proposed three connected contributions. First, it clarified inherited sensitivity as a guiding horizon of interpretation rather than as an already completed mechanism. Second, it made explicit that realized response may serve as a common analytical language across distinct dynamical domains. Third, it showed that threshold selection belongs to the logic of evaluation itself and may therefore be graded or context-dependent.

At the same time, the limits of the present work must remain explicit. The paper does not propose new synchronization equations, a new linear-response theorem, or a completed physical mechanism of inherited sensitivity. Its formal constructions are intentionally minimal and illustrative. The strongest explicit derivation appears in the two-oscillator case, while the network and mediated cases remain more schematic and interpretive.

This point is especially important for the use of Kubo-like or susceptibility-based notation. In the present framework, such notation functions as a comparative and structurally illuminating template. It should not be read as implying that every nonlinear, adaptive, or mediated system considered here is already covered by one universal response theorem. [5]

A similar caution applies to the mediated configurations discussed in Section 7. The present paper does not claim priority over the known literature on relay, remote, or related forms of mediated synchronization. Its claim is narrower: to provide a broader graded response-language within which such configurations may be interpreted as organized cases of mediated realized response. [8–10]

The originality claimed here therefore lies neither in inventing thresholds as such nor in discovering already known mediated phenomena. It lies instead in the explicit cross-domain synthesis of inherited sensitivity as a guiding horizon, realized response as an operational category, and differentiated thresholding as part of the analytical logic of evaluation. In this sense, the work is best understood as a philosophical-physical synthesis equipped with minimal formal illustrations rather than as a completed predictive theory.

The most important consequence of this position is methodological. If the same response may remain latent, measurable, effective, persistent, or reorganizationally significant depending on context, then a binary language of interaction is often insufficient. What the present paper seeks to contribute is a disciplined way of speaking about these different regimes without multiplying ontological categories unnecessarily.

10. Conclusion

The present paper continued the sequence established by the Preface and Papers I–III by focusing on one consequence already implicit in the earlier framework: the threshold of realized response need not always be treated as a single fixed criterion. Once realized response is distinguished from both intrinsic sensitivity and dynamically effective interaction, the threshold may be understood as a criterion of relevance tied to the purpose of evaluation rather than as a universal boundary of sensitivity itself.

On this basis, the paper introduced a graded and context-dependent threshold structure and illustrated it across three connected minimal settings: a direct two-oscillator model, a candidate dynamical-network formulation, and a minimal mediated chain. In each case, the same response-based logic remained visible, while the specific criterion of relevance shifted with the analytical level at issue.

The direct oscillator case provided the clearest explicit threshold relation. The network case showed why thresholding may become intrinsically multi-level once state, output, adaptation, and reorganization are analytically separated. The mediated case then extended the same logic further by distinguishing latent channels from effective pathways and by showing how graded thresholds may also organize mediated regimes.

The paper therefore does not conclude with a new physical law, but with a clarified analytical proposal. Its central claim is that inherited sensitivity, realized response, differentiated thresholding, and mediated response can be brought into one coherent language without erasing the differences between the systems to which that language is applied.

If this proposal is useful, its future value will lie less in claiming universal closure than in guiding further work: more specific mathematical models, more explicit criteria of persistence and organization, numerical studies, and eventual empirical tests. In that sense, the present paper should be read as a structured step in an ongoing philosophical-physical research program rather than as its terminus.

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